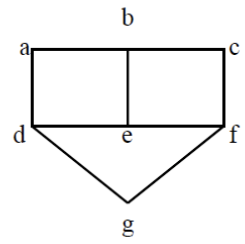
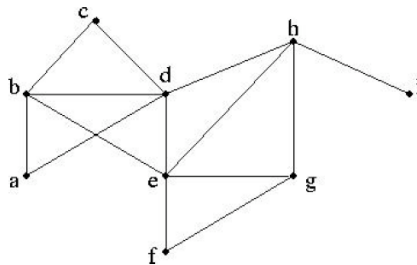


Foundations of Query Languages
Summerterm 11
 Discussion by 08.06.2011

3. Hypergraphs and Treewidth

Exercise 1 (Treewidth)

What is the treewidth of the following graphs? Give a tree decomposition for each graph whose width is the treewidth.



Exercise 2 (Treewidth)

Recall that a graph is 3-colourable if each of its vertices can be coloured either red, green, or blue, such that the endpoints of no edge have the same colour. Consider the class 3COL of 3-colourable graphs. Does this class have bounded or unbounded treewidth? Explain why.

Exercise 3 (Treewidth)

A feedback vertex set of a graph is a set of vertices whose removal would make the graph acyclic. The feedback number $fn(G)$ of a graph G is the minimum cardinality of a feedback vertex set of G .

- a) Show that whenever a class of graphs has bounded feedback number, then also its treewidth is bounded.
- b) Show that the contrary does not hold. That is, there exists a class of graphs of bounded treewidth having unbounded feedback number.

Exercise 4 (Acyclicity)

A property P of graphs (hypergraphs) is called *fully hereditary* if whenever a graph (hypergraph) G has P , then every subgraph (subhypergraph) G' of G obtained from G by eliminating vertices or (hyper)edges also has P . For example, the 3-colourability of graphs is a hereditary property, while being connected (i.e., consisting of a single component) is not hereditary.

- a) Is $treewidth \leq k$ a fully hereditary property? Explain why, or give a counterexample.
- b) Is *acyclic* a fully hereditary property? Explain why, or give a counterexample.